HOSSAM GHANEM

(18) 8.1 Integration By Parts(B)

Example 1 Evaluate the following integral $\int (\sin^{-1} x)^2 dx$

Solution

$I = \int (\sin^{-1} x)^2 dx$

$u = (\sin^{-1} x)^2 \qquad\qquad dv = dx$

$du = 2(\sin^{-1} x) \cdot \dfrac{1}{\sqrt{1-x^2}}\, dx \qquad\qquad v = x$

$I = uv - \int v\, du$

$I = x(\sin^{-1} x)^2 - \int \dfrac{2x}{\sqrt{1-x^2}} \sin^{-1} x\ dx$

$I_1 = \int \dfrac{2x}{\sqrt{1-x^2}} \sin^{-1} x\ dx$

$\int \dfrac{f^{\backslash}(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

$u = \sin^{-1} x \qquad\qquad dv = \dfrac{2x}{\sqrt{1-x^2}} dx$

$du = \dfrac{1}{\sqrt{1-x^2}}\, dx \qquad\qquad v = -2\sqrt{1-x^2}$

$I_1 = -2\sqrt{1-x^2} \sin^{-1} x + \int 2\ dx$

$= -2\sqrt{1-x^2} \sin^{-1} x + 2x + c_1$

$I = x(\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + c$

Example 2 Evaluate the integral $\int 2x\ \sec^{-1} x\, dx$ 14 Nov. 1998

Solution

$I = \int 2x\ \sec^{-1} x\, dx$

$u = \sec^{-1} x \qquad\qquad dv = 2x\, dx$

$du = \frac{1}{x\sqrt{x^2-1}}\, dx \qquad\qquad v = x^2$

$I = uv - \int v\, du$

$I = x^2 \sec^{-1} x - \int \dfrac{x}{\sqrt{x^2-1}}\, dx$

$I = x^2 \sec^{-1} x - \dfrac{1}{2}\int \dfrac{2x}{\sqrt{x^2-1}}\, dx$

$= x^2 \sec^{-1} x - \dfrac{1}{2} \cdot 2\sqrt{x^2-1} + c$

$= x^2 \sec^{-1} x - \sqrt{x^2-1} + c$

Example 3

Evaluate the integral $\int x(\ln x)^2\, dx$ 25 Dec. 2001

Solution

$I = \int x(\ln x)^2\, dx$

$u = (\ln x)^2 \qquad dv = x dx$

$du = \frac{2}{x} \ln(x)\ dx \qquad v = \frac{1}{2}x^2$

$I = uv - \int v\, du$

$I = \frac{1}{2}x^2 \ln(x)^2 - \int x\ \ln x\, dx$

$I_1 = \int x\ \ln x\, dx$

$u = \ln x \qquad dv = x\ dx$

$du = \frac{1}{x}\ dx \qquad v = \frac{1}{2}\ x^2$

$I_1 = \frac{1}{2}\ x^2 \ln x - \frac{1}{2}\int x\ dx$

$= \frac{1}{2}\ x^2 \ln x - \frac{1}{4}x^2 + c_1$

$I = \frac{1}{2}x^2 \ln x^2 - \frac{1}{2}\ x^2 \ln x + \frac{1}{4}x^2 + c$

Example 4

Evaluate the integral $\int \frac{\ln(x+1)}{\sqrt{x+1}}\, dx$ 9 May 1997

Solution

$I = \int \frac{\ln(x+1)}{\sqrt{x+1}}\, dx$

$u = \ln(x+1) \qquad dv = \frac{1}{\sqrt{x+1}} dx$

$du = \frac{1}{x+1}\ dx \qquad v = 2\sqrt{x+1}$

$I = uv - \int v\, du$

$I = 2\sqrt{x+1}\ \ln(x+1) - 2\int \frac{1}{\sqrt{x+1}} dx$

$\frac{1}{x+1} \cdot 2\sqrt{x+1}$

$= \frac{1}{\sqrt{x+1}\cdot\sqrt{x+1}} \cdot 2\sqrt{x+1} = \frac{2}{\sqrt{x+1}}$

$= 2\sqrt{x+1}\ \ln(x+1) - 4\sqrt{x+1} + c$

Example 5

Evaluate the integral $\int e^{\sqrt{x}}\, dx$ 38 July 2005

Solution

$I = \int e^{\sqrt{x}}\, dx$

$t = \sqrt{x} \quad \rightarrow \quad t^2 = x \quad \rightarrow \quad 2t\, dt = dx$

$I = \int 2t\ e^t\, dt$

$u = 2t \qquad dv = e^t\, dt$

$du = 2dt \qquad v = e^t$

$I = uv - \int v\, du$

$I = 2t\ e^t - 2\int e^t\, dt$

$= 2t\ e^t - 2\ e^t + c$

$= 2\sqrt{x}\ e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

Example 6

Evaluate the integral $\int x^5 e^{-x^3}\, dx$ 42 Dec 2006 A

Solution

$I = \int x^5 e^{-x^3}\, dx$

$t = x^3 \quad \rightarrow \quad dt = 3x^2\, dx \quad \rightarrow \quad \frac{1}{3} dt = x^2\, dx$

$I = \int x^3 e^{-x^3} \cdot x^2 dx$

$I = \int \frac{1}{3} t\ e^{-t}\, dt$

$u = \frac{1}{3} t \qquad dv = e^{-t}\, dt$

$du = \frac{1}{3} dt \qquad v = -e^{-t}$

$I = uv - \int v\, du$

$I = \frac{-1}{3} t\ e^{-t} + \frac{1}{3}\int e^{-t}\, dt$

$= \frac{-1}{3} t\ e^{-t} - \frac{1}{3} e^{-t} + c$

$= \frac{-1}{3} x^3 e^{-x^3} - \frac{1}{3} e^{-x^3} + c$

Example 7 Evaluate the integral $\int \sqrt{x}\cos\sqrt{x}\,dx$ 10 August 1997

Solution

$$I=\int \sqrt{x}\cos\sqrt{x}\,dx$$

$$t=\sqrt{x}\quad\rightarrow\quad t^2=x\quad\rightarrow\quad 2t\,dt=dx$$

$$I=2\int t^2\cos t\ dt$$

$u=t^2$ $\qquad dv=\cos t\ dt$

$du=2tdt$ $\qquad v=\sin t$

$$I=uv-\int v\,du$$

$$I=2t^2\sin t-\int 2t\sin t\ dt$$

$$I_1=\int 2t\sin t\ dt$$

$u=2t$ $\qquad dv=\sin t\ dt$

$du=2dt$ $\qquad v=-\cos t$

$$I_1=-2t\ \cos t+2\int\cos t\ dt$$

$$=-2t\ \cos t+2\sin t+c_1$$

$$\therefore I=2t^2\sin t+2t\ \cos t-2\sin t+c$$

Example 8 Suppose that f is continuously differentiable on $[\,0\,,1\,]$ with $f(1)=-3$ and $\int_0^1 x\sqrt{1-f(x)}dx=5.$ Find $\int_0^1\frac{x^2f'(x)}{\sqrt{1-f(x)}}dx.$ (3 pts)

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Solution

$$I=\int_0^1 x\sqrt{1-f(x)}dx$$

$u=\sqrt{1-f(x)}$ $\qquad dv=x\,dt$

$du=\dfrac{-f'(x)}{2\sqrt{1-f(x)}}tdt$ $\qquad v=\dfrac{1}{2}x^2$

$$I=uv-\int v\,du$$

$$I=\left[\frac{1}{2}x^2\sqrt{1-f(x)}\right]_0^1+\frac{1}{4}\int_0^1\frac{x^2f'(x)}{\sqrt{1-f(x)}}dx$$

$$5=\left[\frac{1}{2}(1)^2\sqrt{1-f(1)}-0\right]+\frac{1}{4}\int_0^1\frac{x^2f'(x)}{\sqrt{1-f(x)}}dx$$

$$5=\left[\frac{1}{2}\sqrt{1-(-3)}-0\right]+\frac{1}{4}\int_0^1\frac{x^2f'(x)}{\sqrt{1-f(x)}}dx$$

$$\frac{1}{4}\int_0^1\frac{x^2f'(x)}{\sqrt{1-f(x)}}dx=5-1$$

$$\int_0^1\frac{x^2f'(x)}{\sqrt{1-f(x)}}dx=16$$

Homework

1	Evaluate the following integral	$\int \tan^{-1} 3x\, dx$	22 December 2000
2	Evaluate the following integral	$\int x \tan^{-1} x^2\, dx$	31 December 2003
3	Evaluate the following integral	$\int \sec^{-1} x\, dx$	
4	Evaluate the following integral	$\int \ln x\, dx$	
5	Evaluate the following integral	$\int (\ln x)^2\, dx$	34 July 2004
6	Evaluate the following integral	$\int x^2 (\ln x)^2\, dx$	36 June 2005
7	Evaluate the following integral	$\int x^3 e^{x^2}\, dx$	
8	Evaluate the following integral : ($3\frac{1}{2}$ points)	$\int x^3 (\ln x)^2\, dx$	50 Dec. 15, 2009
9	Use integration by parts to show that [2 marks]	$\int \frac{(1-\ln x)}{x^2} dx = \frac{\ln x}{x} + C$	52 July 24, 2010
10	(3 pts.) Evaluate the following integral	$\int x^2 \sinh x\, dx$	53 11 Dec. 2010
11	Evaluate the following integral	$\int \sqrt{x} \tan^{-1} \sqrt{x}\, dx$	54 12/05/2011
12	Evaluate the following integrals. (3 pts)	$\int \cos(\ln x)\, dx$	55 July 23 , 2011
12	Use integration by parts to evaluate the integral	$\int f(x) dx$ where $f(x) = \int_1^x \frac{\sin t}{t} dt$	54 12/05/2011
13	Evaluate the integral	$\int \arcsin \sqrt{x}\, dx$	5 May 1996